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**Article**

# Time scale for cold-air pool breakup by turbulent erosion

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**Abstract**

Turbulent erosion has been proposed as a major mechanism for removing wintertime cold-air pools (CAPs) from basins and valleys. The time scales involved in this erosion process, which are of great interest for winter weather forecasting, have not been studied systematically in the past. In this short contribution, a semi-analytical model is developed to estimate the time required for the dissipation of cold air pools from above by downward micro-scale turbulent erosion for different wind speeds aloft, different static stabilities inside the cold pool, and for different basin cross sections. The calculations show that micro-scale turbulent erosion is a rather slow process and that the erosion rate decreases rapidly with time as static stability increases in the capping inversion at the top of the cold pool. The rate of erosion is determined mainly by wind speed above the CAP and the temperature inversion strength inside the CAP; it is less sensitive to the shape of the topography. Shallow CAPs of a few tens of meters in depth with a weak inversion may be removed in a matter of hours if winds aloft are sufficiently strong to initiate and maintain turbulent mixing. It is unlikely, however, that deeper CAPs with a moderate to strong inversion can be destroyed by micro-scale turbulent erosion unless combined with other regional and synoptic-scale processes that produce larger-scale turbulent mixing.

**Zusammenfassung**

Erosion durch Turbulenz ist schon lange als wesentlicher Prozess bei der Zerstörung von winterlichen Kaltluftseen in Becken und Tälern betrachtet worden. Der Zeitraum dieses Erosionsprozesses, der besonders wichtig für Vorhersagen im Winter ist, ist bis jetzt jedoch noch nicht systematisch untersucht worden. In diesem Beitrag wird ein semi-analytisches Modell beschrieben, das die benötigte Zeit für die Auflösung eines Kaltluftsees durch mikroskalige turbulente Erosion von oben abschätzt. Dieses Modell berücksichtigt die Windgeschwindigkeit oberhalb und die Stabilität innerhalb des Kaltluftsees sowie den Querschnitt des Beckens bzw. Tales. Die Berechnungen zeigen, dass mikroskalige turbulente Erosion ein langsamer Prozess ist und dass sich die Erosionsrate im Laufe der Zeit stark verlangsamt, wenn die statische Stabilität in der abgrenzenden Inversion oberhalb des Kaltluftsees zunimmt. Die Erosionsrate wird vor allem von der Windgeschwindigkeit oberhalb des Kaltluftsees und vom Temperaturgradienten innerhalb des Sees bestimmt. Der Querschnitt des Beckens bzw. Tales spielt dabei eine weniger wichtige Rolle. Ein flacher Kaltluftsee von einigen Metern (<50 m) mit einem schwachen Temperaturgradienten kann durch mikroskalige turbulente Erosion in einigen Stunden zerstört werden, wenn die Windgeschwindigkeit oberhalb des Sees ausreichend ist, um turbulente Durchmischung zu initiieren bzw. aufrechtzuhalten. Es ist unwahrscheinlich, dass ein tiefer Kaltluftsee mit einem mäßigen bis starken Temperaturgradienten allein durch mikroskalige turbulente Erosion zerstört werden kann, außer dass regionale oder synoptische Prozesse vorhanden sind, die größere Wirbel verursachen.

## 1 Introduction

A cold air pool (CAP) is an accumulation of cold air in a basin or valley (relative to the air above the valley) that is characterized by a persistent temperature inversion. CAPs are especially prevalent in the winter in basins or valleys having poorly developed along-valley wind systems. High static stability in CAPs can trap air overnight or, in mid winter, for many days or even weeks, allowing pollutants, clouds and moisture to build up. For urbanized basins, air pollution can accumulate to unacceptably high levels in CAPs that last for many days (REDDY et al., 1995). CAPs can also affect the valley or basin population by producing hazardous episodes of persistent freezing rain, drizzle or fog, interfering with air and ground transportation (SMITH et al., 1997). Understanding the mechanisms leading to CAP formation and destruction is a problem of practical as well as scientific interest. Several mechanisms have been proposed. In the warm season, diurnal CAPs form on near-calm nights by radiative cooling; they dissipate the next morning when convection begins after sunrise (PETKOVŠEK, 1985; WHITEMAN and MCKEE, 1982). During the cold season, insolation alone may be insufficient to dissi-

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pate CAPs and several other physical processes have been linked to their breakups including cold air advection aloft (ZHONG et al., 2001), frontal passages (WHITEMAN et al., 2001), air drainage from valleys (ZÄNGL, 2002), and turbulent erosion at the top of the pool (PETKOVŠEK, 1978, 1985, 1992; VRHOVEC and HRABAR, 1996; RAKOVEC et al., 2002). Turbulent dissipation or erosion was first suggested by PETKOVŠEK (1985, 1992) as a mechanism for removing CAPs in a relatively wide basin. Using an analytical approach, he proposed two conditions that are necessary for turbulent erosion to work: first, a strong wind above the cold air pool is required to initiate the erosion process; second, this wind speed must increase continuously to maintain the process. For typical Slovenian basins he found that a wind speed of 7 to 9 m $s^{-1}$ is required for shear-generated turbulent mixing to start to erode the temperature inversion from above. Because the capping inversion at the top of the cold pool tends to strengthen during the erosion process, wind speeds above the cold pool have to increase continuously so that the increase in shear production can compensate the increase in buoyancy consumption and maintain turbulence. This requirement for a continuous increase in wind speed above the CAP was supported by numerical simulations of an idealized basin by RAKOVEC et al. (2002). In their idealized simulations, the wind speed above the CAP stopped increasing after reaching 15 m $s^{-1}$. The erosion of the basin inversion, which started at a wind speed around 7 m $s^{-1}$, ceased after the wind speed stopped increasing. Turbulent dissipation was also investigated by the numerical study of VRHOVEC and HRABAR (1996) as one of three mechanisms for dissipation of deep wintertime CAPs. Their study also confirmed that a wind speed increase is necessary for turbulent erosion to continue. It also pointed out that CAP dissipation could take a long time to complete (up to 11 hours) even with wind speed accelerations as large as 2.5 m $s^{-1}$ $h^{-1}$.

The amount of time needed for turbulent erosion to remove a CAP is of great interest for winter weather forecasting in basins and valleys. In this short contribution, we propose a method for estimating the time required to dissipate CAPs by turbulent mixing from above for different wind speeds, stabilities, and basin cross sections.

## 2 Method

As shown in Fig. 1, we assume that a CAP has an initial depth $h$, a potential temperature difference $\theta_b - \theta_c$ within the pool, and a shallow capping inversion layer (CIL) with a potential temperature difference $\theta_a - \theta_b$. We also assume that wind speed above the top of the CAP is $u$ and within the CAP is zero. The wind speed difference across the top of the CAP produces a vertical wind shear that increases with increasing wind speed aloft. Initially, the strong capping inversion keeps the wind aloft from mixing down into the CAP. As the wind aloft continues to increase, turbulence production produced by vertical wind shear may eventually overcome the turbulence consumption caused by negative buoyancy and dissipation. As a result, a downward turbulent sensible heat flux $\rho c_p \overline{\theta' w'}$ is produced and the process of turbulent erosion begins. When this downward sensible heat flux becomes equal to or greater than the heat deficit in a shallow layer of cold air at the top of the CAP, the cold air layer dissipates by erosion.

We use $S$ to denote the horizontal area at the top of the CAP. As turbulent erosion removes the air layer-by-layer from the top of the cold pool, the top of the cold pool descends and the area $S$ decreases (except in the special situation where the slope angle $\alpha$ is 90°). The total sensible heat transported downward across the area $S$ during a small time period $\Delta t$ can be calculated as

$$\rho\, c_p\, \overline{\theta' w'}\, S\, \Delta t. \tag{2.1}$$

The heat deficit within a shallow layer near the top of the CAP can be estimated using

$$\Delta Q = \rho\, c_p\, \Delta\theta\, \Delta V \tag{2.2}$$

where $\Delta V$ is the volume of the layer.

For this shallow layer of cold air to be removed, the turbulent sensible heat transported downward from the top must equal or exceed the heat deficit of this layer, i.e.,

$$\rho\, c_p\, \overline{\theta' w'}\, S\, \Delta t \geq \rho\, c_p\, \Delta\theta\, \Delta V. \tag{2.3}$$

The time required for removing this shallow layer of cold air, therefore, is

$$\Delta t \geq \frac{\rho\, c_p\, \Delta\theta\, \Delta V}{\rho\, c_p \overline{\theta' w'}\, S} = \frac{\Delta\theta\, \Delta V}{\overline{\theta' w'}\, S}. \tag{2.4}$$

The downward turbulent sensible heat flux $\overline{\theta' w'}$ can be described using K-theory (LUMLEY and PANOFSKY, 1964) as

$$\overline{\theta' w'} = -K_h \frac{\partial\theta}{\partial z} \approx -K_h \frac{\Delta\theta}{\Delta z} \tag{2.5}$$

Substituting (2.5) into (2.4) yields a simple equation for $\Delta t$,

$$\Delta t \geq \frac{\Delta\theta\, \Delta V}{\overline{\theta' w'}\, S} = \frac{\Delta\theta\, \Delta V}{K_h \frac{\Delta\theta}{\Delta z}\, S} = \frac{\Delta V\, \Delta z}{K_h\, S}. \tag{2.6}$$

Here, $\Delta\theta$ cancels in the equation if we assume that the depth of the layer removed is the same as the depth of the layer over which the turbulent sensible heat flux is calculated using the bulk approximation based on $K$ theory.

The area $S$ and volume $\Delta V$ are geometric parameters determined by the shape of the basin. For a circular basin with a floor radius of $r_{flr}$ and a constant slope angle $\alpha$ the horizontal area at height $z$ is

$$S(z) = \pi\, r^2(z) \tag{2.7}$$

where

$$r(z) = r_{flr} + z/\tan\alpha \tag{2.8}$$

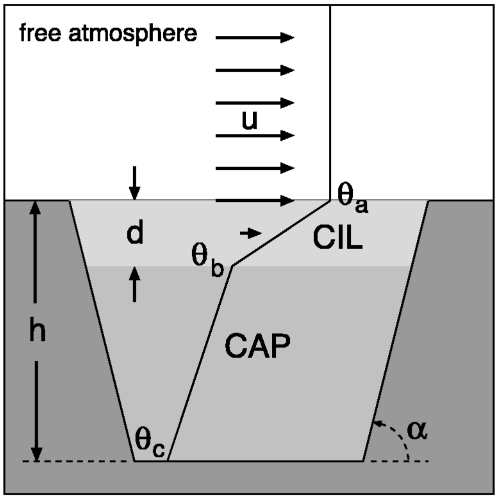


**Figure 1:** Sketch illustrating a cold-air pool in a basin period.

is the radius of the circular basin at height $z$.

The volume $\Delta V$ is the volume of the frustum of a cone and is given by

$$\begin{aligned} \Delta V &= \frac{\pi}{3}\Delta z\,[r^2(z+\Delta z/2) \\ &+ r^2(z-\Delta z/2) \\ &+ r(z+\Delta z/2)\cdot r(z-\Delta z/2)]. \end{aligned} \tag{2.9}$$

Normally the slope of the terrain $\alpha$ is much smaller than 90° and both $S$ and $\Delta V$ decrease as the top of the cold pool sinks. These terrain parameters become independent of height only when $\alpha = 90°$. In this case, $r(z) \equiv r_{flr}, S(z) \equiv \pi\, r_{flr}^2$, and $\Delta V \equiv \pi\, r_{flr}^2 \Delta z$ and Equation (2.6) then becomes

$$\Delta t \geq \frac{\Delta V\ \Delta z}{K_h\ S} = \frac{\Delta z^2}{K_h}. \tag{2.10}$$

The eddy diffusivity $K_h$ needed in Equation (2.6) or its special form, Equation (2.10), is determined from BLACKADAR's (1979) formulation

$$K = l^2\,\frac{\partial u}{\partial z}\left[1 - \frac{Ri}{Ri_{cr}}\right]^{\frac{1}{2}} \tag{2.11}$$

where $Ri = \frac{g}{\theta}\frac{\partial\theta}{\partial z}/(\frac{\partial u}{\partial z})^2$ is the gradient Richardson number, and the critical Richardson number $Ri_{cr}$ is approximately 0.25 (TAYLOR, 1931; BUSINGER et al., 1971). The quantity $l$ represents the turbulent mixing length scale which, according to BLACKADAR (1962, 1979), is

$$l = \kappa z / \left[1 + \frac{\kappa z}{\lambda_o}\right] \tag{2.12}$$

where $\kappa = 0.35$ is the Von Karman constant, $\lambda_o = 0.0063\, u_*\, f^{-1}$ is the asymptotic value of $l$ for very large $z, f$ is the Coriolis parameter, and $u_*$ is the turbulent friction velocity, which can be parameterized as $u_* = c_d\, u$ where $c_d$ is the drag coefficient. DEARDORFF (1968) showed that $c_d$ is a function of $Ri$, and decreases gradually from $(1+0.07\,u)/1000$ when $Ri = 0$ to 0 when $Ri = Ri_{cr}$.

If the wind speed aloft, the temperature difference across the capping inversion, and the temperature inversion within the CAP are known for a circular basin with a slope angle $\alpha$, Equations (2.6–2.12) allow us to estimate the time interval required for turbulent erosion to remove a thin layer near the top of the CAP. The total time period for the complete removal of the entire CAP is simply the summation of the time intervals, i.e. $T = \sum_i \Delta t_i$.

One can also estimate the minimum wind speed aloft required to initiate the process of turbulent erosion based on the critical Richardson number. Turbulent erosion is enabled when the bulk Richardson number for the capping inversion layer becomes sub-critical, i.e.,

$$R_b = \frac{g}{\theta}\frac{\Delta\theta\ d}{(\Delta u)^2} < 0.25 \tag{2.13}$$

where $d$ is the depth of the CIL (Fig.1). For simplicity, we have assumed calm conditions inside the CAP, and in such case, $\Delta u$ is equivalent to the wind speed aloft, $u$. From Equation (2.13) we obtain

$$u > 2\sqrt{\frac{g}{\theta}\,\Delta\theta\, d} = 2\sqrt{\frac{g}{\theta}\,\frac{\Delta\theta}{d}\,d^2} = 2\,N\,d \tag{2.14}$$

where $N = \sqrt{\frac{g}{\theta}\frac{\Delta\theta}{d}}$ represents the Brunt-Väisälä frequency in the CIL.

## 3 Results

From Equation (2.14), we can estimate the wind speed aloft that will initiate turbulent erosion for different chosen stabilities and capping inversion layer depths. Results are shown in Fig. 2. The minimum wind speed increases with increasing CIL depths and Brunt-Väisälä frequencies or capping inversion strengths. The minimum wind speed is 7 m s$^{-1}$ for N = 0.06 s$^{-1}$ and $\Delta z = 50$ m, increasing to nearly 16 m s$^{-1}$ for N = 0.08 s$^{-1}$ and $\Delta z = 50$ m.

Once the wind speed aloft exceeds the threshold value, turbulent erosion begins. Assuming a CAP with a 500 m depth and a capping inversion layer of 50 m in a circular basin, we calculate, using Equations (2.6–2.12), the time needed to destroy the CAP for six scenarios with different values of basin geometry, stability,

**Table 1:** Parameters assumed in the calculations for the six cases.

| *case* | $\theta_a(K)$ | $\theta_b(K)$ | $\theta_c(K)$ | $u(m s^{-1})$ | $r_{flr}(km)$ | $\alpha$ |
|---|---|---|---|---|---|---|
| 1 | 295 | 290 | 285 | 10 | 10 | 15 |
| 2 | 295 | 290 | 285 | 12 | 10 | 15 |
| 3 | 300 | 290 | 285 | 15 | 10 | 15 |
| 4 | 300 | 290 | 285 | 15 | 10 | 90 |
| 5 | 300 | 290 | 285 | 15 | 0 | 15 |
| 6 | 300 | 290 | 290 | 15 | 10 | 15 |

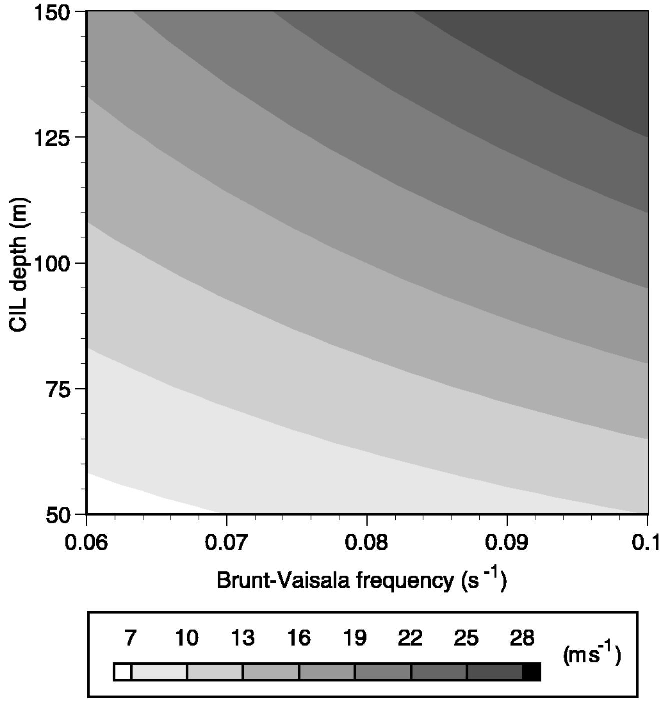


**Figure 2:** Contours of minimum wind speeds above the cold air pool required for initiating turbulent erosion as a function of the depth of the capping inversion layer and the Brunt-Väisälä frequency in the layer.

and wind speed aloft. The parameters used for the six scenarios are summarized in Tab. 1. Cases 1, 2, 3 and 6 assume a circular basin with a flat floor of radius $r_{flr}$ and sidewalls of angle $\alpha = 15°$. Case 5 assumes a V-shaped basin where $r_{flr} = 0$ and Case 4 deals with a special case where the slope angle is 90 degrees, which is close to a U-shaped basin. In all but one case, a potential temperature increase of 5 K is assumed within the CAP with potential temperature on the basin floor $\theta_c = 285$ K and potential temperature at the base of the capping inversion $\theta_b = 290$ K. Case 6 deals with the special case where the stratification within the basin is neutral ($\theta_c = \theta_b = 290$ K). The calculations start from the top of the CAP and progress downward until the entire CAP is removed.

The results of the calculations are given in Fig. 3, which shows the decrease of the CAP top as a function of time for all six cases. A comparison of Cases 1 and 2 reveals the direct influence of wind speed on the rate of turbulent erosion. For the same stability and basin geometry, the descent rate of the CAP top or the erosion rate becomes considerably larger when the wind speed aloft increases from 10 m s$^{-1}$ in Case 1 to 12 m s$^{-1}$ in Case 2. Consequently, the top of the CAP drops about 100 m after 3 days for Case 1, and nearly 400 m for Case 2. A substantial increase in the sinking rate of the CAP top also occurs when wind speed increases from 12 m s$^{-1}$ in Case 2 to 15 m s$^{-1}$ in Case 3, but the difference between Cases 2 and 3 is smaller than that between Cases 1 and 2. This is because the potential temperature difference across the CIL doubled from Case 2 to Case 3, which partially cancels the effect of enhanced wind shear. It is interesting to note that changes in basin geometry (Cases 3–5) result in relatively small changes in the rates of erosion. The CAP top descends at similar rates for the circular basin with flat floor and constant angle sidewalls (Case 3) and for the U-shaped (Case 4) basin, but is somewhat faster for the V-shaped basin (Case 5), indicating that if all other conditions are the same, turbulent erosion is more efficient for a V-shaped basin. Finally, the fastest descent of the inversion top occurs, as anticipated, for the case of neutral stability within the CAP (Case 6). In this case, for the given strength of the capping inversion (10 K) and the wind speed aloft (15 m s$^{-1}$), turbulent erosion generated by the vertical shear of horizontal winds overpowers the inversion to completely remove a 500 m deep CAP in a circular basin in slightly over 30 hours.

## 4 Conclusion

The results indicate that micro-scale turbulent erosion of cold air from the top of a CAP is a rather slow process. The erosion rate depends mostly on the wind speed aloft and the inversion strength and is less sensitive to the shape of the topography. A shallow cold air pool of a few 10 s of meters with a weak inversion could be removed in less than a day if winds aloft were sufficiently strong to initiate and maintain turbulent mixing. It would, however, take several days to break a CAP of a few hundred meters depth with a moderate inversion. For CAPs that are deep or capped by a strong inversion, it is unlikely that micro-scale turbulent erosion alone could destroy them unless the winds aloft were very strong and increased rapidly with time. Such situations are normally associated with synoptic-scale disturbances such as frontal passages. It is, therefore, concluded that micro-scale turbulent erosion can not break up deep wintertime cold air pools unless combined with other larger-scale processes. It is our hope that this theoretical study and our initial conclusions will motivate further field investigations of this phenomenon.

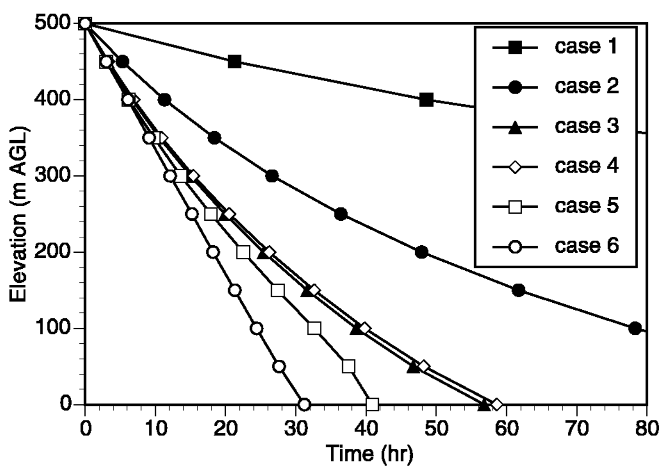


**Figure 3:** The decrease in height of the cold air pool top as a function of time for the 6 different scenarios listed in Tab. 1.

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